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Estimating the Low-Speed Sidewash Gradient on a Vertical Stabilizer

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Nomenclature

b = wingspan
b' = wingtip vortex spacing
C_{Lw} = wing lift coefficient
R_{Aw} = wing aspect ratio
V_z = z component of induced velocity
V_∞ = magnitude of the freestream velocity
x = axial coordinate
$\bar{x}$ = dimensionless axial coordinate, $2x/b$
y = normal coordinate
$\bar{y}$ = dimensionless normal coordinate, $2y/b$
z = spanwise coordinate, measured from the aircraft plane of symmetry
$\bar{z}$ = dimensionless spanwise coordinate, $2z/b$
z' = spanwise coordinate, measured from the centerline midway between the two wingtip vortices
$\bar{z}'$ = dimensionless spanwise coordinate, $2z'/b$
β = sideslip angle
Γ_{wt} = wingtip vortex strength
ε_s = sidewash angle
κ_b = spacing between the wingtip vortices divided by the wingspan
κ_v = ratio of wingtip vortex strength to that of an elliptic wing having the same lift coefficient and aspect ratio
κ_β = sidewash factor
Λ = quarter-chord wing sweep angle

Introduction

THE sidewash induced on a vertical stabilizer by the wingtip vortices shed from the main wing can have a significant effect on the static yaw stability of an airplane. For a vertical stabilizer mounted above the main wing, the sidewash gradient is negative and has a stabilizing effect on the airplane. The sidewash gradient produced by the wingtip vortices can be estimated using a vortex model that was recently presented for predicting the downwash produced by a wing of arbitrary planform.[1] This vortex model, which is shown in Fig. 1, was originally proposed by McCormick[2] for estimating the downwash produced by an elliptic wing.

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Analytical Model

Relative to the coordinate system shown in Fig. 1, the z component of velocity induced by this model of the wingtip vortices, at the arbitrary point in space (x, y, z), is readily found from the Biot–Savart law to be

$$V_z = \frac{\Gamma_{\mathrm{wt}}}{4\pi}\left\{\frac{y}{y^2+\left(z+\frac{1}{2}b'\right)^2}\right.\times\left[1+\frac{x-\frac{1}{2}b'\tan\Lambda}{\sqrt{\left(x-\frac{1}{2}b'\tan\Lambda\right)^2+y^2+\left(z+\frac{1}{2}b'\right)^2}}\right]-\frac{y}{y^2+\left(z-\frac{1}{2}b'\right)^2}\left.\times\left[1+\frac{x-\frac{1}{2}b'\tan\Lambda}{\sqrt{\left(x-\frac{1}{2}b'\tan\Lambda\right)^2+y^2+\left(z-\frac{1}{2}b'\right)^2}}\right]\right\} \tag{1}$$

The wingtip vortex strength is proportional to the product of the wing lift coefficient and airspeed. The wingtip vortex strength and spacing can be evaluated from lifting-line theory.[1] When the traditional sign convention that sidewash is positive from left to right is used, Eq. (1) is combined with lifting-line theory, and the small angle approximation is applied, the sidewash angle can be written as

$$\varepsilon_s = -\frac{V_z}{V_\infty} = \frac{C_{Lw}\kappa_v}{\pi^2 R_{Aw}}\left\{\frac{\bar{y}}{\bar{y}^2+(\bar{z}-\kappa_b)^2}\right.\times\left[1+\frac{\bar{x}-\kappa_b\tan\Lambda}{\sqrt{(\bar{x}-\kappa_b\tan\Lambda)^2+\bar{y}^2+(\bar{z}-\kappa_b)^2}}\right]-\frac{\bar{y}}{\bar{y}^2+(\bar{z}+\kappa_b)^2}\left.\times\left[1+\frac{\bar{x}-\kappa_b\tan\Lambda}{\sqrt{(\bar{x}-\kappa_b\tan\Lambda)^2+\bar{y}^2+(\bar{z}+\kappa_b)^2}}\right]\right\} \tag{2}$$

where the parameters κ_v and κ_b can be evaluated analytically from the known planform shape of the wing using the results presented by Phillips et al.[1]

Equation (2) can be directly applied to determine the sidewash angle only when the sideslip angle is zero and the aircraft plane of

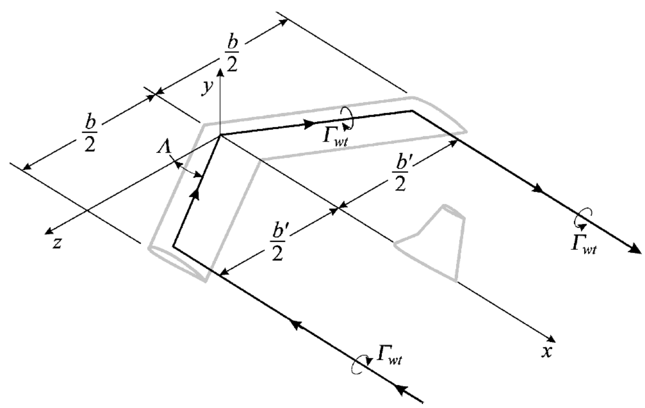

Fig. 1 Vortex model used to estimate the sidewash gradient for the vertical stabilizer.

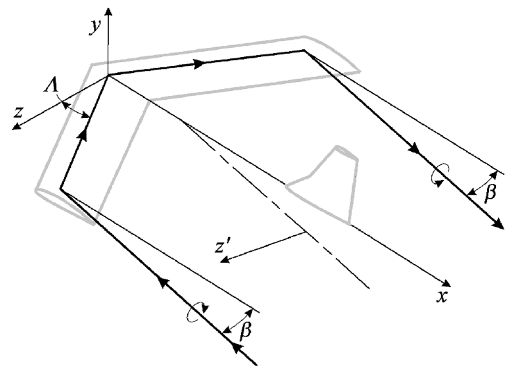

Fig. 2 Effect of sideslip on the position of the wingtip vortices relative to the vertical stabilizer.

symmetry is midway between the two trailing vortices. When the airplane has some component of sideslip, the wingtip vortices are displaced relative to the position of the vertical tail, as shown in Fig. 2. For small sideslip angles, Eq. (2) gives a very close approximation for the sidewash angle, if the z coordinate, measured from the aircraft plane of symmetry, is replaced with a z' coordinate, measured from the centerline midway between the two wingtip vortices, as shown in Fig. 2. When the small angle approximation is used, the z' coordinate is related to the z coordinate according to

$$z'(\beta) = z\cos\beta - \left(x - \tfrac{1}{2}b'\tan\Lambda\right)\sin\beta \cong z - \left(x - \tfrac{1}{2}b'\tan\Lambda\right)\beta$$

or

$$\bar{z}'(\beta) \cong \bar{z} - (\bar{x} - \kappa_b\tan\Lambda)\beta \tag{3}$$

Within this small angle approximation, the sidewash gradient can be written as

$$\frac{\partial\varepsilon_s}{\partial\beta} = \frac{\partial\bar{z}'}{\partial\beta}\frac{\partial\varepsilon_s}{\partial\bar{z}'}\bigg|_{\bar{z}'=\bar{z}} \tag{4}$$

where, from Eq. (2)

$$\begin{aligned}\frac{\partial\varepsilon_s}{\partial\bar{z}'}\bigg|_{\bar{z}'=\bar{z}} = \frac{C_{Lw}\kappa_v}{\pi^2 R_{Aw}}\Bigg\{&\frac{-2\bar{y}(\bar{z}-\kappa_b)}{\left[\bar{y}^2+(\bar{z}-\kappa_b)^2\right]^2}\\ &\times\left[1+\frac{\bar{x}-\kappa_b\tan\Lambda}{\sqrt{(\bar{x}-\kappa_b\tan\Lambda)^2+\bar{y}^2+(\bar{z}-\kappa_b)^2}}\right]\\ &+\frac{2\bar{y}(\bar{z}+\kappa_b)}{\left[\bar{y}^2+(\bar{z}+\kappa_b)^2\right]^2}\\ &\times\left[1+\frac{\bar{x}-\kappa_b\tan\Lambda}{\sqrt{(\bar{x}-\kappa_b\tan\Lambda)^2+\bar{y}^2+(\bar{z}+\kappa_b)^2}}\right]\\ &-\frac{\bar{y}}{\bar{y}^2+(\bar{z}-\kappa_b)^2}\left[\frac{(\bar{x}-\kappa_b\tan\Lambda)(\bar{z}-\kappa_b)}{\left[(\bar{x}-\kappa_b\tan\Lambda)^2+\bar{y}^2+(\bar{z}-\kappa_b)^2\right]^{\frac{3}{2}}}\right]\\ &+\frac{\bar{y}}{\bar{y}^2+(\bar{z}+\kappa_b)^2}\left[\frac{(\bar{x}-\kappa_b\tan\Lambda)(\bar{z}+\kappa_b)}{\left[(\bar{x}-\kappa_b\tan\Lambda)^2+\bar{y}^2+(\bar{z}+\kappa_b)^2\right]^{\frac{3}{2}}}\right]\Bigg\}\end{aligned} \tag{5}$$

and from Eq. (3)

$$\frac{\partial\bar{z}'}{\partial\beta} = -(\bar{x} - \kappa_b\tan\Lambda) \tag{6}$$

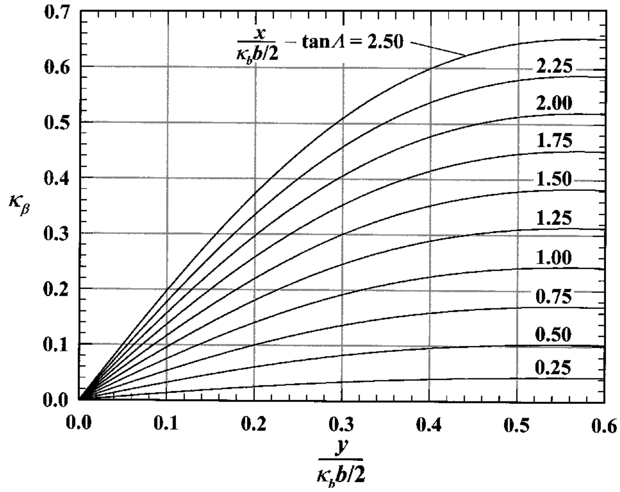

Fig. 3 Effect of vertical stabilizer position on the sidewash gradient in the plane of symmetry.

The sidewash gradient induced at an arbitrary point in space can be estimated by using Eqs. (5) and (6) in Eq. (4). In the plane of symmetry this reduces to

$$\frac{\partial\varepsilon_s}{\partial\beta}(\bar{x},\bar{y},0) = -\frac{\kappa_v\kappa_\beta}{\kappa_b}\frac{C_{Lw}}{R_{Aw}} \tag{7}$$

where

$$\begin{aligned}\kappa_\beta(\bar{x},\bar{y},0) = &\frac{4\bar{y}(\bar{x}-\kappa_b\tan\Lambda)\kappa_b^2}{\pi^2\left(\bar{y}^2+\kappa_b^2\right)^2}\\ &\times\left[1+\frac{\bar{x}-\kappa_b\tan\Lambda}{\sqrt{(\bar{x}-\kappa_b\tan\Lambda)^2+\bar{y}^2+\kappa_b^2}}\right]\\ &+\frac{2\bar{y}\kappa_b}{\pi^2\left(\bar{y}^2+\kappa_b^2\right)}\left[\frac{(\bar{x}-\kappa_b\tan\Lambda)^2\kappa_b}{\left[(\bar{x}-\kappa_b\tan\Lambda)^2+\bar{y}^2+\kappa_b^2\right]^{\frac{3}{2}}}\right]\end{aligned} \tag{8}$$

The sidewash factor κ_β depends on the planform shape of the wing and the position of the tail relative to the wing. However, the planform shape of the wing affects the value of κ_β only through Λ and κ_b. Thus, κ_β is a unique function of $\bar{x}/\kappa_b - \tan\Lambda$ and $\bar{y}/\kappa_b$, as shown in Fig. 3.

It is important to remember the orientation of the coordinate system used in the development of Eqs. (5) and (8). As shown in Figs. 1 and 2, the x–z plane of this coordinate system coincides with the plane of the trailing wingtip vortices, and the x axis is aligned with the equilibrium relative wind. Thus, the y coordinate is the vertical distance measured above the plane of the trailing wingtip vortices. When the downwash from this vortex model was computed according to the relations presented by Phillips et al.,[1] the y coordinate was found to have only a second-order effect on downwash. However, as can be seen from Eqs. (5) and (8), the y coordinate has a first-order effect on the sidewash. Thus, the orientation of the x axis within the aircraft plane of symmetry is critical for predicting the sidewash gradient. This orientation changes with angle of attack as shown in Fig. 4. The y coordinate of any point on the aft vertical stabilizer decreases with increasing angle of attack. From Eq. (5), it is seen that the sidewash on the vertical stabilizer is proportional to the product of the wing lift coefficient and the dimensionless y coordinate $\bar{y}$. As angle of attack is increased, the wing lift coefficient increases but $\bar{y}$ decreases. Thus, for a given wing configuration, the magnitude of the sidewash gradient does not increase linearly with

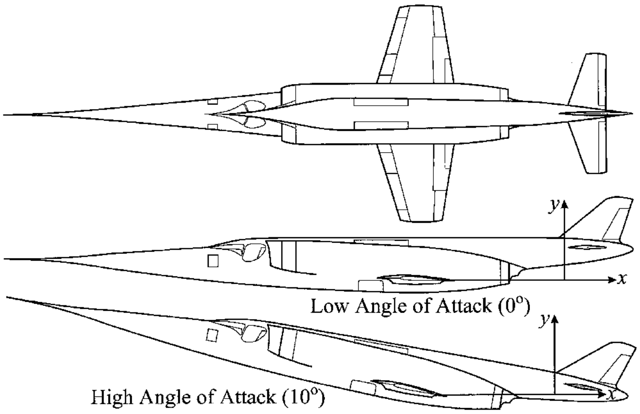

Fig. 4 Effect of aircraft angle of attack on the *y* position of the vertical stabilizer.

lift coefficient. In fact, as the angle of attack increases from zero lift, the magnitude of the sidewash gradient will, at some point, begin to decrease with increasing angle of attack. Ultimately, the sidewash gradient becomes positive and destabilizing when the angle of attack is large enough to put the vertical stabilizer below the wingtip vortices. Normally the wing would stall well before this angle of attack was reached. However, in poststall maneuvers, this can produce an important destabilizing effect.

Conclusions

The sidewash gradient predicted from the present analytical model increases rapidly with decreasing wing span. Thus, aircraft having wings of low aspect ratio produce large sidewash gradients. In addition, because the sidewash gradient is proportional to the wing lift coefficient, slow flight with high-lift devices deployed can also result in a significant increase in the sidewash gradient induced on the vertical stabilizer.

The present model also points out that the sidewash gradient on the vertical stabilizer increases rapidly with its distance aft of the wingtips. Because the yaw stability derivative increases with the product of the area of the vertical stabilizer and its distance aft of the center of gravity, an important consideration in tail design is the tradeoff between the tail area and length required to attain the desired level of yaw stability. As the tail is moved aft, the required size of the vertical stabilizer is decreased, which reduces tail weight and drag. However, moving the tail aft increases the weight and drag associated with the structure supporting the tail. Thus, there is an optimum in the tradeoff between tail area and length. Because the sidewash gradient on a high vertical tail becomes more stabilizing with increasing tail length, sidewash moves this optimum in the direction of smaller tails placed farther aft of the center of gravity. As the wingspan is decreased, this effect becomes more important. However, moving the vertical stabilizer farther aft of the wingtips will aggravate the destabilizing effect of sidewash in poststall maneuvers.

The proposed model does not account for the effects of the fuselage on the sidewash gradient. The fuselage can significantly alter the flowfield induced by the wingtip vortices. Depending on fuselage geometry, the sidewash gradient induced on a vertical stabilizer by the wingtip vortices can be altered by as much as 30% as a result of flow around the fuselage.

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